Chapter 8

Conclusions

In this Thesis I have presented an overview of the field of research on electronic correlations in nanostructures. I have started with a general introduction to quantum dots and compared their fundamental properties to those of natural atoms. I have shown that both classes of systems - QDs and natural atoms - possess many common characteristics to the extent that the QDs are often called "artificial atoms". However, these two systems exhibit several important differences, and perhaps the most important of them in the context of correlations is the strong atomic quantisation of single-particle (orbital) energy contrasted with much weaker, and tunable, QD confinement. This soft QD confinement allows to build confined systems, in which the quantisation of orbital energy introduces the same energy scale as Coulomb carrier-carrier interactions. For this reason, theoretical and experimental study of properties of QDs allows to gain insight into the correlated behaviour of strongly interacting carriers.

To introduce the reader into the subject of QD physics and technology, I have presented the most important methods of fabrication of QDs. I have divided these methods into three groups. In the first technique the lateral confinement is created electrostatically by means of gates, locally depleting the two-dimensional gas. Such dots allow to study electronic systems only. Another technique, utilising the Stranski-Krastanow phase tran-

sition in the epitaxial growth of heterostructures, yields dots with relatively small sizes, capable of confining both electrons and holes. Both types of carriers can be also confined in quantum rings, fabricated by lithography and etching. I have also summarised the most important experimental techniques used to probe the properties of many-particle systems confined in QDs.

But the main focus of this thesis is a fundamental research into the properties of correlated systems confined in nanostructures. I started my detailed discussion of these properties by defining mathematical methods and computational tools used in my work. I described the single-particle spectra of several idealised confinement potentials, approximating the actual QD systems, and used these spectra as an input in the many-body problem.

The problem of many interacting particles confined in the lateral confining potential of nanostructures was defined in the language of fermion creation and annihilation operators. To be complete, this formulation requires the single-particle spectrum, established before, and also the Coulomb scattering matrix elements. These elements have been calculated analytically in the case of the two-dimensional parabolic lateral confinement.

Further I have discussed the possible mathematical and computational methods that could be used to solve the many-body problem. I needed a method capable of treating the correlation effects on equal footing with the direct and exchange Coulomb interactions. I have reviewed three such methods, the exact diagonalisation technique, the spin density functional theory and the quantum diffusion Monte Carlo approach. However, for my research I have chosen the exact-diagonalisation configuration-interaction method, because only this method allows to obtain *exact* description of the system, albeit only for relatively small number of particles.

In my detailed description of the configuration-interaction approach, I have described how the symmetries of the system could be exploited in order to reduce the size of the many-particle basis set and achieve greater computational accuracy. I have also shown

that special tools are needed in order to diagonalise these optimised, but still very large Hamiltonian matrices. My tool of choice was the conjugate gradient minimisation technique combined with the spectrum folding method. By use of this computational tool I could find the ground and several lower excited states of matrices with size of order of $10^5 \times 10^5$ and larger, i.e., objects so large that they could not be stored all at once in the computer memory.

Having completely defined the many-body problem and the methods of solving it I have analysed the correlation effects in parabolic quantum dots with or without the magnetic fields, containing electrons only, and coupled quantum dot and quantum ring systems confining both electrons and holes. My presentation of this material was comprised of a series of articles, integrated into the text.

In my analysis of parabolic many-electron quantum dots, I have carefully distinguished the effects due to electronic correlations from those brought about by the direct and exchange Coulomb terms. I showed that the latter terms favourize more spin-polarised systems, and are counteracted by correlations whose essence lies in configuration mixing. This is due to the fact that the low-spin subspaces usually have much larger basis sets than the high-spin ones, and therefore acquire a correlation advantage additionally lowering their energy. This correlation advantage can lead not only to shifts in phase boundaries of the system, but also to the appearance of entirely new phases whose presence is verified by experimental data. In particular, I have demonstrated the composite nature of the second spin-flip transition in the eight-electron droplet. I have shown that in the evolution of the system with the magnetic field, the region of stability of the second-spin-flip configuration is preceded by strongly correlated, spin-unpolarised phases. This prediction was confirmed experimentally in the high source-drain voltage tunnelling spectroscopy. This was - to the best of my knowledge - the first experimental observation of correlated states in electronic QDs with known and controlled number of electrons in the presence of a magnetic field.

Further I have analysed the electron-hole systems. I have shown that the Coulomb

interactions between an electron and a hole confined in a vertically coupled double-disk structure lead to the entanglement of states of the two carriers, which can be viewed as a particular manifestation of correlations. I have also shown that the electron-hole correlations strongly influence the behaviour of an exciton and a charged exciton on a quantum ring, and may lead to the suppression of Aharonov-Bohm oscillations of their energy. As a result, the photoluminescence spectrum of the charged complex is dominated by the energy of the final state electron. This allows to observe the Aharonov-Bohm oscillations of a single electron directly in the photoluminescence spectra.